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A Hierarchy Hypothesis

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Abstract

Why attraction and repulsion between likes should not enjoy equal status in nature is considered. By postulating a hierarchy of isolated systems of finite radii whose associated charges form a geometric series with enormous imaginary common ratio, and by identifying a "universe" (the content of an infinite cosmos within a Hubble radius of an observer), an electron, and a neutrino as three consecutive members of the hierarchy (in fact the only three observable because of the uncertainty principle), it is possible to treat gravitational and electromagnetic phenomena as perfectly analogous and complementary for the overall structure of the cosmos. An isolated system behaves, from an external viewpoint, as an elementary particle, and from an internal viewpoint, as a universe. Remarkable relationships between physical constants emerge.

We consider the relationship between the electromagnetic and gravitational fields in a flat space-time. Elsewhere it will be argued that the geometry of space-time of necessity is open to free choice; inevitably an observer relies on the timing of light signals in order to record events [the observations open to him being described in some detail by Milne (1935)], and the coordinates which he assigns to the events depend on the adoption of an arbitrary convention, either with regard to the velocity of light propagation (in which case the geometry is fixed) or alternatively with regard to the geometry of the space-time on which the events are mapped (in which case the velocity of light is fixed). With the choice of flat space-time, "radar distances" are in fact given by nonholonomic expressions.

One aspect of the relationship between the gravitational and electromagnetic fields can be appreciated by considering an arbitrary static distribution of point charges e_k at points $\mathbf{r}_k$, for which the potential field ϕ' is given by

$$\nabla^2 \phi' = -4\pi \sum_k e_k \delta(\mathbf{r} - \mathbf{r}_k) \tag{1}$$

The energy density U' in the electrostatic field derived from ϕ' now acts as a source for a gravitational field, for which the potential function ϕ'' satisfies

$$\nabla^2 \phi'' = -4\pi i G^{1/2} [(\boldsymbol{\nabla}\phi')^2 + (\boldsymbol{\nabla}\phi'')^2] / (8\pi K c^2) \tag{2}$$

Corresponding to electromagnetic energy density U' is inertial mass density U'/c^2 and gravitational mass density U'/Kc^2, where K is a constant with dimensions (usually given the value unity by choice of units). The energy density in the gravitational field itself is also a source in (2). The imaginary character of gravitational fields expresses that like masses attract, whereas like charges repel.

By means of the substitution

$$\chi = \exp(\eta\phi), \qquad \eta = \pm iG^{1/2}/(2Kc^2) \tag{3}$$

we can write (2) in the form

$$\nabla^2\chi = -8\pi\eta^2 U'\chi, \qquad U' = (\nabla\phi')^2/8\pi \tag{4}$$

Two particular cases will be of interest.

Let U' be the energy density in the electrostatic field of a point charge q. Then $U' = q^2/(8\pi r^4)$, and (4) has the solution

$$\chi = A \cosh(a/r + B), \qquad a = \tfrac{1}{2}Gq^2/(Kc^2) \tag{5}$$

which yields

$$\phi'' = (iq/a)\ln[A \cosh(a/r + B)]$$

$$\mathbf{E}'' = -\nabla\phi'' = (iq\mathbf{r}/r^3)\tanh(a/r + B) \tag{6}$$

Thus $E \simeq iG^{1/2}m/r^2$ for $r \gg a$, where B is chosen so that $\tanh B = G^{1/2}m/q$, and $E \simeq iq/r^2$ for $r \ll a$. Thus, in the limit of small r the energy density in the gravitational field (negative) becomes sufficient to cancel that in the electrostatic field. The transition from weak to strong gravity occurs for $r \sim a$.

Secondly, let U' equal the constant mean density of electromagnetic energy associated with a constant mean density of mass ρ_u throughout the universe. We put $U' = K\rho_u c^2$ into (4), and obtain the solution

$$\chi = r^{-1}[C \sinh(\lambda r + D] \qquad \lambda^2 = 2\pi G\rho_u/(Kc^2) \tag{7}$$

For $D = 0$ and $\lambda r \ll 1$, we have the approximate expression

$$\chi = (C/r)(\lambda r + \lambda^3 r^3/3! + \cdots) \tag{8}$$

which yields

$$\phi'' = 2iKc^2G^{-1/2}[\ln(C\lambda) + \tfrac{1}{4}r^2/R_u{}^2 + \cdots] \tag{9}$$

where we define $R_u{}^2 = 3/(2\lambda^2)$. In the potential field (9), obtained without imposing any cutoff on ρ_u, matter will expand. A particle of mass M which is released from rest at the origin will have acquired kinetic energy $\frac{1}{2}Mv_g{}^2K$ at radial distance r, where $v_g/c = r/R_u$.

When the charges in (1) have motions, a real magnetic field will arise in accordance with Maxwell's equations. This will produce a mass current and hence an imaginary magnetic field, assuming that the generalization of (2) is analogous to that of (1). The generalization of (2), at least in the first instance, has been given by Hund (1948).

Still further refinements will be required in order to take into account non-uniformity of the ether when the convention of flat space-time is adopted. In fact Dicke (1957) has used a scalar refractive index for the ether as gravitational field variable, with success in accounting for the classic general relativistic effects. Probably inclusion of all refinements would give equations formally equivalent to those of general relativity (where space-time is curved as a consequence of the convention of constant light velocity).

The development of a theory of gravitation, step by step along the above lines, lends support to an old suggestion due to Wilson (1921) that the gravitational interaction between pieces of matter is a high order electromagnetic interaction independent of the sign of charge—in fact a gradient of self-energy caused by variation in the polarizability of the vacuum near to matter. But there is an equally compelling alternative attitude. Why should nature prefer attraction between likes to repulsion between likes; in other words, why should gravitation and electromagnetism, as phenomena, enjoy unequal status in the natural world? Might we not be dealing with complementary aspects of a single complex field? At first sight it might seem that the two viewpoints are mutually exclusive. In this paper our aim is to show that by introducing a hierarchy hypothesis the two viewpoints can in fact be reconciled.

According to the hierarchy hypothesis there exist within an infinite cosmos a hierarchy of isolated systems, each specified by a charge (from which can be derived a radius, a time, and a mass). The charges, which are alternately real and imaginary (and hence assign equal status to attraction and repulsion between likes), are assumed to form an infinite geometric series with common ratio $i\beta$, where $i = (-1)^{1/2}$:

$$\cdots q_{+2}, iq_{+1}, q_0, iq_{-1}, q_{-2}, \cdots \qquad (10)$$

Associated with a charge q_n is a mass m_n, a radius a_n, and a time T_n, where

$$q_n^2 = -Gm_n^2, \ \tfrac{1}{2}q_n^2/a_n = Km_nc^2, \ T_n = a_n/c \qquad (11)$$

The constants for the nth isolated system of the hierarchy are obtained from the constants of the $(n-1)$th system by applying the scaling factor $i\beta$ to each of the dimensions, charge (or mass), length, and time.

So enormous is β that only three of the isolated systems will ever affect the measurements which an observer can make (in principle). These are a "universe" (charge, $iG^{1/2}M_u$), an electron [charge, $(\hbar c)^{1/2}$], and a neutrino (charge $iG^{1/2}\mu$). Here a "universe" is defined as the content of the cosmos within a Hubble radius R_u of any particular observer. We assign to a universe the properties of an elementary particle—that is, an electron or neutrino. With regard to the electron, it is assumed that the Coulomb charge e results from the more fundamental bare charge $(\hbar c)^{1/2}$ by renormalization of charge. In the case of the neutrino, a finite rest mass μ is assigned to the particle; this raises no problems because it will turn out that the neutrino rest energy $K\mu c^2$ is related to the age R_u/c of a universe by Heisenberg's uncertainty formula, so that energies smaller than $K\mu c^2$ cannot be measured.

At first sight the hierarchy hypothesis may seem too fantastic to be reasonable. However, it is the only way to avoid certain basic asymmetries in the "real" as opposed to the observable world (the model which is closest to "reality" is that which can not merely account for all observed phenomena, but which has the minimum of ad hoc assumptions). Within a "universe", and outside an electron, asymmetry is inevitable. Gravitational charge $iG^{1/2}m$ enjoys a status different from charge q; thus a universe is a region where matter prevails over antimatter and where mass is positive. Implied is the existence also of "antiuniverses" where the reverse will obtain. The unique direction of time is another manifestation of asymmetry in the observable world.

Universes and antiuniverses are assumed to play the role of particles and antiparticles in a superuniverse—in fact, superelectrons and superpositrons. It is assumed that the elementary particles of a universe are electrons and positrons, that all matter, including the mesons, nucleons, and baryons, will prove in the ultimate analysis to be systems of only electrons and positrons (Browne, 1966). The superuniverse, in turn, will be the elementary particle of an even greater isolated system, and so on ad infinitum. The hierarchical scheme will continue downward as well as upward. The electron, from an internal point of view, becomes a subuniverse based on the neutrino as elementary particle. Each isolated system appears from an internal point of view to be a universe and from an external point of view an elementary particle.

Applying (11) to the electron, one obtains for the electron radius

$$a_0 = \tfrac{1}{2}K^{-1}(G\hbar/c^3)^{1/2} = 8.07 \times 10^{-34} \text{ cm} \tag{12}$$

This result would also emerge from (5) on putting $q = (\hbar c)^{1/2}$, which clarifies its significance. Applying (11) to a universe we find

$$M_u = 2KR_uc^2/G \tag{13}$$

If ρ_u is the mean density of mass in a universe, we may write $M_u = (4\pi/3)R_u{}^3\rho_u$ and (13) yields the well known result

$$(2\pi/3)G\rho_u(R_u/c)^2 = K \tag{14}$$

The scaling factor β can now be evaluated. Using (13) we obtain

$$\beta = G^{1/2}M_u/(\hbar c)^{1/2} = R_u/a_0 = 2.13 \times 10^{61} \tag{15}$$

Here we have adopted $R_u = 1.72 \times 10^{28}$ cm, a theoretical value which, however, is very close to the most recent observational estimate (see below). It also follows that $M_u = 4.64 \times 10^{56}$ g and $\rho_u = 1.09 \times 10^{-29}$ g cm^{-3}.

Since we also have that

$$(\hbar c)^{1/2}/(G^{1/2}\mu) = \beta \tag{16}$$

it follows that

$$(R_u/c)(2K\mu c^2) = \hbar \tag{17}$$

which is the uncertainty relation mentioned above and proposed previously (Browne, 1962). By limiting the span over which measurements are possible the uncertainty principle ensures that our observations reveal asymmetry at a basic level. Our theoretical value for R_u implies $\mu = 1.00 \times 10^{-66}$ g.

The rather precise value which has been adopted for R_u stems from the following argument. We consider the distance from a singularity where there is a balance between the internal field of a universe and the external field of a neutrino, both centered on the singularity. For the external field of the neutrino we have $iG^{1/2}\mu/r^2$, and for the internal field of the universe (9) gives $iKc^2 G^{-1/2} R_u^{-2} r$. We assume that the fields balance at $r = r_0 = e^2/(Kmc^2)$ $= 2.82 \times 10^{-13}$ cm (the so-called classical electron radius). This assumption leads to the following relationship between physical constants:

$$r_0{}^3 = G\mu R_u{}^2/(Kc^2) = 2a_0{}^3\beta \tag{18}$$

where use has been made of (17). On the basis of a somewhat different argument Rosen (1967) (see also Harris, 1969) has arrived at an order of magnitude relation of type (18). From (18) one deduces $R_u = 1.72 \times 10^{28}$ cm, which corresponds to a Hubble constant of $H = c/R_u = 53.5$ km/sec Mpc^{-1}. This is remarkably close to the most recent observational estimate, 55 km/sec Mpc^{-1} (Sandage, 1973).

More remarkable still is the possibility, at least in principle, of deriving the fine structure constant, $\alpha = e^2/(\hbar c) = (137.03602)^{-1}$. The procedure is to use β^2 to impose a natural cutoff for the logarithmically divergent integrals in quantum electrodynamics. In lowest-order perturbation theory the electron possesses a self-mass Δm due to emission and absorption of virtual photons, where (Heitler, 1954)

$$\frac{\Delta m}{m} = \frac{\alpha}{2\pi} \int_0^{\epsilon_m} \frac{2\epsilon^2 - 1}{\epsilon^3}\, d\epsilon \simeq \frac{3\alpha}{2\pi} \ln \epsilon_m \tag{19}$$

ϵ being the invariant energy of the intermediate state in units of the electron's rest energy. By limiting the maximum intermediate state energy ϵ_m the logarithmic divergence of $\Delta m/m$ can be prevented. But when $\Delta m/m$ becomes of order unity, higher-order terms in the perturbation theory expansion can no longer be neglected. We would like to assign to the electron a bare mass μ which is extremely small, and then by means of a nonperturbative calculation impose the condition that $\Delta m/m = 1$ for $\epsilon_m = \beta^2$. If we impose this condition on (19), then only a rather crude result can be expected; this is

$$\beta \sim \exp(\tfrac{1}{3}\pi/\alpha) = 2.10 \times 10^{62} \tag{20}$$

We may note also that exp $(1/\alpha) = 3.27 \times 10^{59}$. Thus values for β are obtained which are not greatly different from (15). In principle renormalization of the charge of an electron from $(\hbar c)^{1/2}$ to e by imposition of a cutoff also should yield a prediction for β in terms of the fine structure constant.

The space-time in Einstein's theory which corresponds to the cosmological field (9) is that of de Sitter. Introducing coordinates which comove with the matter, de Sitter space-time has the Robertson metric,

$$ds^2 = c^2\, d\tau^2 - \exp(2c\tau/R_u)(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \qquad (21)$$

Consider, now, the emission of successive wave crests at times τ_1 and $\tau_1 + \delta\tau_1$, the waves being received at τ_2 and $\tau_2 + \delta\tau_2$. The first crest travels a distance $c\delta\tau_1 \exp(-c\tau_1/R_u)$ before the second crest is emitted, whilst the second crest travels a distance $c\delta\tau_2 \exp(-c\tau_2/R_u)$ after the first is absorbed. The comoving distance coordinate r between source and absorber does not change, so that the above distances must be equal. That is, the wavelengths at emission and reception are equal, but the frequencies bear the ratio

$$\nu_2/\nu_1 = \delta\tau_1/\delta\tau_2 = \exp[-c(\tau_2 - \tau_1)/R_u] \qquad (22)$$

What is of particular interest is that the Hubble red shift is then a change of frequency at constant wavelength for a reference system with respect to which matter is at rest. We have to consider the change of frequency of radiation which propagates in a medium with time-dependent refractive index. The frequency of refractive index variation is $c/R_u (\equiv \omega_u)$. Assuming nonlinearity, we must look to the type of parametric frequency conversion familiar in nonlinear optics, in which energy in a "pump" oscillation is transferred to "signal" and "idler" oscillations when the sum of the frequencies of the latter equals the frequency of the pump oscillation. In our case the radiation field of frequency ω provides the pump oscillation; the signal oscillation occurs at frequency ω_u and the idler then has frequency $\omega - \omega_u$, which is the frequency of the radiation slightly red-shifted. The idler oscillation then becomes the pump for further parametric frequency conversion, giving a gradual frequency decrease with distance propagated.

From (17) we see that $\hbar\omega_u = 2K\mu c^2$, suggesting that the signal oscillation is gravitational radiation with constant quantum energy $\hbar\omega_u$ $(\equiv \epsilon)$ (the graviton). The graviton represents the minute rest energy of a neutrino pair. The gradual loss of photon energy to the graviton field can be understood if Planck's radiation oscillators are quantized as states of a fundamental oscillator with energy ϵ equal to that of the graviton:

$$\hbar\omega = (n + \tfrac{1}{2})\epsilon, \qquad \epsilon = \hbar\omega_u = 2K\mu c^2 \qquad (23)$$

The photon can now be considered as a field of gravitons. Because only $\Delta n = \pm 1$ transitions are allowed the photon is constrained to lose gravitons gradually, explaining why the red-shift occurs gradually.

If one assumes that the successive loss of gravitons from the photon field is a scattering process, then for a medium with N scatterers per unit volume the number of gravitons lost from a photon field n per unit time will be

$n\phi_0 cN$, where ϕ_0 is the scattering cross section. The energy lost per unit distance is $\epsilon n\phi_0 N$, and hence we can write

$$\hbar d\omega = \epsilon n\phi_0 N\, dr = \hbar\omega N\phi_0\, dr \tag{24}$$

This represents the Hubble red shift if $N\phi_0 = 1/R_u$. The scatterers in the intergalactic medium might be gravitons, or possibly neutrinos. If we assume $N = K\rho_u c^2/\epsilon = \rho_u/2\mu$, then by use of (17) and (14) we find that $\phi_0 = (8\pi/3)a_0{}^2$, which is of the expected order of magnitude (Wheeler, 1962). In extensive regions of above average graviton or neutrino density anomalous red shifts would be produced.

Thus, if the Hubble red shift in comoving coordinates is to be interpreted as a parametric frequency conversion process, then we must associate with the medium a natural frequency ω_u whose significance is connected with radiation–neutrino interactions. The hierarchy hypothesis would lead us to expect an ether which, at its coarsest level, might behave as a superfluid of electrons, and, at a sublevel, as a superfluid of neutrinos. Strong phenomenological support for the superfluid ether can be found in the remarkable parallel between the double-barrier Josephson effect (Jaklevic et al., 1964) and the Aharanov-Bohm effect (Aharanov & Bohm, 1959; Weisskopf, 1960).

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